

Comment on "On quantum plasma: A plea for a common sense [Europhys. Lett., 99 25001 (2012)]"

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Abstract –We point out flaws in the work of Vranjes *et al.* (EPL **99** 25001, 20012), and present correct criteria for quantum plasmas.

In a recent letter, Vranjes *et al.* [1] are giving the impression that numerous theories for quantum plasmas are new, which have been flourishing only during the past few years. This is misnomer, since the importance of the degeneracy of electrons was recognized through the pioneering works of Fowler and Chandrasekhar [2]. During the early phase of quantum mechanics, Madelung has developed the quantum fluid theory for electrons by using the Schrödinger equation. About sixty years ago, many distinguished physicists [3, 4] laid down the foundation to collective interactions in dense quantum plasmas. In fact, an early experiment [5] in 1956 had already provided an experimental evidence for the quantum feature of the electron plasma waves in solids, where the electrons are in a degenerate state, and thereby supporting the linear theory of Klimontovich and Silin [3] and Bohm and Pines [4]. Recently, there has been a surge in investigating numerous nonlinear processes [6–9] in quantum plasmas, where the electron degeneracy and quantum effects are shown to play a crucial role. Furthermore, applications of collective interactions in quantum plasmas rest on exploring new physics of stimulated scattering of intense laser beams off quantum electron plasma oscillations [10] and quantum free-electron lasers in the x-ray regime, and the discovery of novel attractive force [11] that can bring ions closer at atomic dimensions in order for the magneto-inertial confinement fusion to work in a highly compressed high-energy solid density plasma.

Our objective here is to demonstrate that Vranjes *et al.*'s criteria for the applicability of quantum plasmas are

totally misleading and invalid/erroneous. First of all, we recall that the plasma state occurs in the form of an ionized gas through which electricity can flow. The Saha's ionization criterion dictates that in order for the plasma to form one must have sufficiently high degree of ionization. Thus, ordinary air at room temperature is not in the plasma state, since the fractional ionization n_i/n_n is ridiculously low $\sim 10^{-122}$. The fractional ionization, however, increases with the increase of the gas temperature by heating, so that the electrons are liberated from neutral atoms and they are a component of the electron-ion plasma which can be fully or partially ionized. In a fully ionized classical electron-ion plasma, the thermal de Broglie wavelength $\lambda_B = \hbar/\sqrt{2\pi m_e k_B T_p}$, which characterizes the spatial extension of the probability density of the electrons, is much less than the average inter-electron distance $d \sim (3/4\pi n_0)^{1/3}$, where \hbar is Planck's constant divided by 2π , m_e the electron mass, k_B the Boltzmann constant, T_p the plasma temperature, and n_0 the average electron number density. In low-temperature laboratory plasmas with $T_p = 300$ degrees Kelvin, we then have $n_0 \ll 10^{18} \text{cm}^{-3}$, and there is no need to consider the electron degeneracy effect, and one uses the classical theory for ideal plasmas with the Maxwell-Boltzmann distribution for non-degenerate electrons and ions. However, the situation is different for metallic conduction electrons where (due to their high mobility) they are degenerate with moderate density of $3 \times 10^{22} \text{cm}^{-3}$ at room temperature. Thus, in order for the quantum effect to become important, the thermal de Broglie wavelength of quantum particles must

be $\leq d$, and the Landau length $\lambda_l = e^2/k_B T_p \geq d$, with e being the magnitude of the electron charge. It turns out that, $d \geq \hbar/\sqrt{2\pi m_e k_B T_p}$ and $k_B T_p \geq e^2/d$ are the appropriate criteria to define the range of a quantum plasma with degenerate electrons.

It must be stressed that in a quantum plasma, degenerate electrons follow the Fermi-Dirac distribution function, which gives an expression that relates the Fermi electron energy $E_F = k_B T_F$ and n_0 . One has [6] $T_F = (\hbar^2/2k_B m_e)(3\pi^2 n_0)^{2/3}$, which reflects the quantum nature of the Fermi electron temperature T_F through \hbar . Vranjes *et al.* [1] have confused the Fermi-temperature, T_F , with the plasma temperature T_p . About five years ago, Glenzer *et al.* [10] reported observations of the electron plasma oscillations in a solid density plasma (with the peak electron number density $\sim 3 \times 10^{23} \text{cm}^{-3}$ and the equilibrium electron and ion temperatures of 12 eV ($T_p \sim 1.4 \times 10^5$ degrees Kelvin), which is different from the Fermi electron temperature for metals (of order $10^4 - 10^5 \text{K}$), by using collective x-ray scattering techniques. Thus, the experiments of Glenzer *et al.* [10] have unambiguously demonstrated the quantum dispersive effects associated with quantum statistical pressure and the electron recoil effects in solid density laboratory plasmas.

On the other hand, in his Nobel Prize winning paper, Chandrasekhar [2] presented the famous pressure law for degenerate electrons, which reads $P_c = (m_e^4 c^5 / 24 \pi^2 \hbar^3) [R(2R^2 - 3)\sqrt{1 + R^2} + 3 \sinh^{-1} R]$, with $R = (P_{Fe}/m_e c) = (n_0/n_c)^{1/3}$ where P_{Fe} is the electron relativistic Fermi-momentum and $n_c = m_e^3 c^3 / 3 \pi^2 \hbar^3 \simeq 5.9 \times 10^{29} \text{cm}^{-3}$. The generalized Chandrasekhar degeneracy pressure P_c reduces to $P_n \approx (\pi^2/5)(3/\pi)^{2/3}(\hbar^2/m_e)n_0^{5/3}$ and $P_u \approx (\pi/4)(3/\pi)^{1/3}\hbar c n_0^{4/3}$ in non-relativistic and ultra-relativistic cases obtained in $R \ll 1$ and $R \gg 1$ limits, respectively. The pressure for degenerate electrons and the plasma temperature are related through $n_0 k_B T_p = P_c$.

It seems that Vranjes *et al.* [1] deduced the electron degeneracy criterion for non-relativistic quantum plasmas from $n_0 k_B T_F = P_n$, which simply reflects the relationship between T_F and n_0 and cannot be regarded as correct criterion to define quantum plasmas. The correct criteria for the electron degeneracy effect in quantum plasmas come from $d \leq \lambda_B, \lambda_l$, which yield $n_0^{1/3} \leq (3/4\pi)^{1/3} \sqrt{2\pi m_e k_B T_p} / \hbar$, $(3/4\pi)^{1/3} e^2 / k_B T_p$. The latter hold for quantum plasmas in solids [5], in high-energy density compressed plasmas [10], giant planetary systems [12], and in compact astrophysical objects (e.g. white dwarf stars [2]). Finally, we note that Vranjes *et al.*'s [1] have presented an incorrect expression for T_{deg} that is proportional to $n_0^{3/2}$, and declare it a meaningful criterion for the electron degeneracy in quantum plasmas. To conclude, we can say that the figure and the table displayed in Ref. [1] are fallacious, and are of no use for defining the regimes of quantum plasmas.

Figure 1 exhibits the regions where quantum plasmas are in nonrelativistic and relativistic electron degeneracy

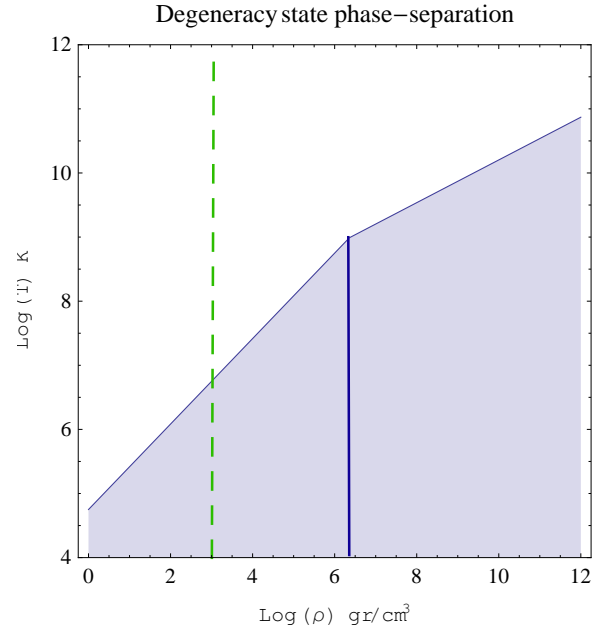


Fig. 1: Degeneracy phase-separation diagram with the shaded area corresponding to the electron degeneracy. The vertical dashed line indicates the density above which the plasma is completely pressure ionized.

states (the shaded area), which are separated into non-relativistic and relativistic degeneracy regimes at a critical mass-density $\log(\rho_{cr}) \simeq 6.34$ defined by $P_n = P_u$. The non-shaded area corresponds to the classical ideal gas phase.

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